DESIGN OF FUNCTIONALLY GRADED COMPOSITES FOR STRENGTH AND STIFFNESS

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The design of graded composite materials for optimal strength and stiffness is addressed. We consider graded composites made up of two elastic phases occupying the structural domain Ω. The volume of the structure is denoted by V. The stress inside a composite with characteristic length scale ε is denoted by σ^ε. The equivalent stress is given by the quadratic function $\Pi(\sigma^\varepsilon)$. The strength of each phase is described by failure probabilities. The failure probabilities of each phase are assumed to be given by the Weibull distributions

$$P_i^\varepsilon = 1 - \exp(-\int_{\Omega_i^\varepsilon} c_i(\sigma^\varepsilon)\,dx),\ \ i = 1, 2. \tag{1}$$

The integral is taken over the part of Ω occupied by the i^{th} phase denoted by Ω_i^ε. The Weibull concentration function in phase i is $c_i(\sigma^\varepsilon) = \gamma_i \Pi(\sigma^\varepsilon)^{p_i}$, where p_i is the Weibull's modulus in the i^{th} phase. Here γ_i and p_i are parameters that characterize each phase. Recent results in homogenization [1] show that

$$\lim_{\varepsilon\to 0} P_i^\varepsilon \le 1 - \exp(-\gamma_i \|f^i(\sigma^M)\|_\infty^{p_i} V),\ \ i = 1, 2. \tag{2}$$

Here the homogenized or macroscopic stress is denoted by σ^M and $f^i(\sigma^M)$ is called the macrostress modulation function and measures the amplification or diminution of σ^M inside the i^{th} phase by the microstructure, [1]. Roughly speaking the macrostress modulation is calculated at each point by applying the macrostress to the local composite microgeometry in a neighborhood of the point and evaluating the maximum equivalent stress over the i^{th} phase in that neighborhood. The sup norm of the macrostress modulation over Ω is $\|f^i(\sigma^M)\|_\infty^{p_i}$ and it is sensitive to the appearance of local stress concentrations. The upper bound on the failure probability given in (2) goes to 1 when $f^i(\sigma^M)$ diverges. Motivated by these results we present numerical methods for the design of graded elastic materials for optimal stiffness while at the same time keeping the probabilities to failure within acceptable values. We design according to constraints on the upper bound given in (2). In this way our designs are conservative but are self consistent in that they are based upon a stress analysis carried out before significant stress redistribution occurs due to nonlinear effects. Numerical computations are given for shafts subject to torsion and for graded laminates.

References

[1] R. Lipton, "Assessment of the local stress state through macroscopic variables," *Philosophical Transactions of the Royal Society, Mathematical, Physical and Engineering Sciences*, to appear 2003.

[3] R. Lipton and M. Stuebner "Design of graded composites for strength and stiffness," in preparation.